Achieving Optimal Revenue with Enhanced Competition

Linda Cai

General Exam at Princeton University
Overview of My Research

Auction Design

• Prove that simple auction can achieve 99% of optimal revenue with constant enhanced competition [EC21]
  
  *Joint work with Raghuvansh Saxena*

• Implementation in advised strategies: a new solution concept for self interested behavior when being truthful is NP-hard [ITCS20]
  
  *Joint work with Clayton Thomas and Matt Weinberg*

• Repeated auction design for buyers using no regret learning algorithms
  
  *Joint work with Matt Weinberg, Evan Wildenhain and Shirley Zhang*

Stable Matching

• A simple proof for short-side advantage in random matching markets [SOSA21]
  
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- Maximizing revenue is hard when there are multiple items
- Enhanced competition: can a simple auction achieve the optimal revenue by recruiting more bidders
- Our result: simple auction can achieve 99% of the optimal revenue with constant enhanced competition
Revenue Maximizing Auction

**Combinatorial auction:** $n$ bidders, $m$ items.

- Each bidder $i$ has valuation function $v_i : 2^m \to \mathbb{R}^+$. 
- Bidders participate in some (possibly interactive) protocol.
- Auctioneer awards the set of items $S_i$ to bidder $i$, charges price $p_i$. 

Bidder Goal: Maximizes (expected) utility = $v_i(S_i) - p_i$. 

Auctioneer Goal: Maximizes (expected) revenue = $\sum_i p_i$. 

Bidder goal different from auctioneer goal, how can the auctioneer predict bidder behavior?

**Truthful Auction (Informal)**

An auction is **truthful** if it is in the bidder's best interest to behave truthfully (e.g. bidding their own value). 

Auctioneer Constraint: Use truthful auctions.
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Revenue Maximizing Auction: Our Setting

\[ v_i \approx D_j = \times D_j \]

...\[ v_i(\cdot) = 5 \]
\[ v_i(\cdot) = 6 \]
\[ v_i(\cdot, \cdot) = 11 \]

\( n \) i.i.d additive bidders, \( m \) items
Revenue Maximizing Auction: Our Setting

\( v_i \sim \mathcal{D} = \times_j \mathcal{D}_j \)

\( n \) i.i.d. additive bidders, \( m \) items
Revenue Maximizing Auction: Our Setting

\[ v_i \sim D = \times_j D_j \]

\[ v(\text{apple}) = 5 \]

\[ v(\text{banana}) = 6 \]

\[ v(\text{apple, banana}) = 11 \]

\[ n \text{ i.i.d additive bidders, } m \text{ items} \]
Revenue Maximizing Auction: Our Setting

\[ v_i \sim D = \times_j D_j \]

\[ v(-0x0) = 5 \]

\[ v(-0x0, -0x0) = 11 \]

\[ n \text{ i.i.d additive bidders, } m \text{ items} \]
Revenue Maximizing Auction: Our Setting

\[ \nu_i \sim \mathcal{D} = x_j \mathcal{D}_j \]

\[ \nu(\text{apple}) = 5 \]

\[ \nu(\text{banana}) = 6 \]

\[ \nu(\text{apple}, \text{banana}) = 11 \]

\[ n \text{ i.i.d additive bidders, } m \text{ items} \]
Revenue Maximizing Auction: Our Setting

\( v_i \sim D = \times_j D_j \)

We want to maximize revenue using truthful auctions.

\( v(\text{apple}) = 5 \)
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Revenue Maximizing Auction: Our Setting

$v_i \sim D = \times_j D_j$

$v(\text{apple}) = 5$

$v(\text{banana}) = 6$

We want to maximize revenue using **truthful** auctions.

**Bayesian Incentive Compatible:** the bidder’s expected utility is maximized by behaving truthfully when other bidders also behave truthfully.
Maximizing Revenue: Single Item Setting

One item, one bidder:

Myerson

The optimal auction is a posted price auction.

Example: \( v \in U[0,1] \)

Revenue from selling item at price \( p \):

\[ p \cdot \Pr[\text{value} \geq p] = p(1-p) \]

Optimal auction: sell item at price \( p = 1/2 \)
One item, one bidder:

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Optimal auction: sell item at price $p = 1/2$
Maximizing Revenue: Single Item Setting

One item, *multiple* bidders:

\[
\text{Let } F \text{ be the c.d.f of } D, \text{ let } f \text{ be the p.d.f of } D \quad (\text{each bidder } i \text{'s value } v_i \sim D) \\
\text{Myerson The optimal auction maximizes the expected Myerson virtual value } \phi(v_i) = v_i - 1 - F(v_i) f(v_i) \text{ of the bidder that gets the item.} \\
\text{When the virtual value function is regular, the optimal auction is second price auction with reserve.}
\]
Maximizing Revenue: Single Item Setting

One item, multiple bidders:

Let $F$ be the c.d.f of $D$, let $f$ be the p.d.f of $D$

(each bidder $i$’s value $v_i \sim D$)

**Myerson**

The optimal auction maximizes the expected **Myerson virtual value**

$$
\varphi(v_i) = v_i - \frac{1-F(v_i)}{f(v_i)}
$$

of the bidder that gets the item.
Maximizing Revenue: Single Item Setting

One item, multiple bidders:

Let $F$ be the c.d.f of $D$, let $f$ be the p.d.f of $D$

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**Myerson**

The optimal auction maximizes the expected Myerson virtual value

$$\varphi(v_i) = v_i - \frac{1-F(v_i)}{f(v_i)}$$

of the bidder that gets the item.

When the virtual value function is *regular*, the optimal auction is *second price auction* with reserve.
Maximizing Revenue: Multiple Item Setting

**Example:** two items, one bidder

\[ v_1 = 1 \text{ w.p. } 1/2 \text{ and } v_1 = 2 \text{ w.p. } 1/2 \]

\[ v_2 = 1 \text{ w.p. } 3/4 \text{ and } v_2 = 4 \text{ w.p. } 1/4 \]
Maximizing Revenue: Multiple Item Setting

Example: two items, one bidder

\[ v_1 = 1 \text{ w.p. } \frac{1}{2} \text{ and } v_1 = 2 \text{ w.p. } \frac{1}{2} \]
\[ v_2 = 1 \text{ w.p. } \frac{3}{4} \text{ and } v_2 = 4 \text{ w.p. } \frac{1}{4} \]

Optimal deterministic auction: selling separately

Revenue: 2
**Example:** two items, one bidder

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**Optimal deterministic auction:** selling separately

Revenue: 2

Menu \((p_1, p_2)\): get item 1 w.p. \(p_1\), get item 2 w.p. \(p_2\)
**Example:** two items, one bidder

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**Optimal deterministic auction:** selling separately

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Menu \((p_1, p_2)\): get item 1 w.p. \(p_1\), get item 2 w.p. \(p_2\)

**Randomize auction:** \((1, \epsilon)\) with price \(2 + \epsilon\)

\((\epsilon, 1)\) with price \(4 + \epsilon\)

\((1, 1)\) with price \(6 - 3\epsilon\)
Example: two items, one bidder

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Menu \((p_1, p_2)\): get item 1 w.p. \(p_1\), get item 2 w.p. \(p_2\)

Randomize auction: \((1, \epsilon)\) with price \(2 + \epsilon\) preferred by \(v_1 = 2, v_2 = 1\)

\((\epsilon, 1)\) with price \(4 + \epsilon\)

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Maximizing Revenue: Multiple Item Setting

Example: two items, one bidder

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Menu \((p_1, p_2)\): get item 1 w.p. \(p_1\), get item 2 w.p. \(p_2\)

**Randomize auction:** (1, \(\epsilon\)) with price \(2 + \epsilon\) preferred by \(v_1 = 2, v_2 = 1\)

\((\epsilon, 1)\) with price \(4 + \epsilon\) preferred by \(v_1 = 1, v_2 = 4\)

\((1, 1)\) with price \(6 - 3\epsilon\) preferred by \(v_1 = 2, v_2 = 4\)

Revenue:

\[
\frac{1}{2} \cdot \frac{3}{4} (2 + \epsilon) + \frac{1}{2} \cdot \frac{1}{4} (4 + \epsilon) + \frac{1}{2} \cdot \frac{1}{4} (6 - 3\epsilon) = 2 + \frac{\epsilon}{8}
\]
Revenue optimal auctions are messy when $m > 1$:

- **(Non-monotonicity)** It might get less revenue from bidders with higher values. [HR15]

- **(Randomness)** It might sell “lottery tickets” for sets of items. [Tha04, MV07, Pav11, DDT17]

- **(Intractability)** It might present uncountably infinite number of “lottery tickets”. [HN13, DDT14]
Approximating Revenue Is Possible But With Unsatisfactory Constants

<table>
<thead>
<tr>
<th>Paper</th>
<th>$n$</th>
<th>$m$</th>
<th>Bidder Type</th>
<th>Approximation Ratio</th>
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</thead>
<tbody>
<tr>
<td>[BILW14]</td>
<td>$n = 1$</td>
<td>arbitrary</td>
<td>additive</td>
<td>6</td>
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<td>[CDW16]</td>
<td>arbitrary</td>
<td>arbitrary</td>
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<td>[GK16]</td>
<td>arbitrary</td>
<td>arbitrary</td>
<td>additive, regular</td>
<td>200 (prior-independent auction)</td>
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<td>[CZ17]</td>
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<td>arbitrary</td>
<td>XOS</td>
<td>268</td>
</tr>
<tr>
<td>[CZ17]</td>
<td>arbitrary</td>
<td>arbitrary</td>
<td>subadditive</td>
<td>$\log m$</td>
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<td>...</td>
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Yes! With enhanced competition!
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**Enhanced Competition**

Find number of bidders \(n' > n\) where a simple auction with \(n'\) bidders (almost) match the revenue of the optimal auction with \(n\) bidders.
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Find number of bidders \(n' > n\) where a simple auction with \(n'\) bidders (almost) match the revenue of the optimal auction with \(n\) bidders.

- **Motivation:** Instead of spending effort designing the optimal (or close to optimal) auction, spend effort recruiting bidders!
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**Enhanced Competition**

Find number of bidders \(n' > n\) where a simple auction with \(n'\) bidders (almost) match the revenue of the optimal auction with \(n\) bidders.

- **Motivation:** Instead of spending effort designing the optimal (or close to optimal) auction, spend effort recruiting bidders!

- **Focus of our paper:** constant enhanced competition – Is it possible to use only \(n' = O(n)\) bidders?
Progress on Constant Enhanced Competition

For which $n$, $m$ is constant enhanced competition enough to get almost full revenue?
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Progress on Constant Enhanced Competition

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[FFR18, BW19]: $n = 1$ or $n \gg m$

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Progress on Constant Enhanced Competition

For which \( n, m \) is constant enhanced competition enough to get almost full revenue?

- **[BK96]:** \( m = 1 \)
- **[FFR18, BW19]:** \( n = 1 \) or \( n \gg m \)
- \( n \ll m: O(n \log(\frac{m}{n})) \)
Progress on Constant Enhanced Competition

For which $n, m$ is constant enhanced competition enough to get almost full revenue?

$\frac{n}{m} \ll n \approx m: O\left(n \log\left(\frac{m}{n}\right)\right)$

Our result: for all $n, m$

- $[FFR18, BW19]: n = 1$ or $n \gg m$
- $[BK96]: m = 1$
Theorem 1 (informal)
A simple auction with $n' = O(n/\epsilon)$ bidders can obtain a $(1 - \epsilon)$ fraction of the optimal revenue with $n$ bidders.
Our Results

Theorem 1
Let $\epsilon > 0$ and $n' = O(n/\epsilon)$. At least one of the following hold:

1. A $(1 - \epsilon)$-fraction of the optimal revenue with $n$ bidders is obtained by a second price auction with $n'$ bidders.

2. A simple auction (either selling the items separately or a second price auction with an entry fee) with $n'$ bidders generates more revenue than the optimal auction with $n$ bidders.
Theorem 1
Let $\epsilon > 0$ and $n' = O(n/\epsilon)$. At least one of the following hold:

1. A $(1 - \epsilon)$-fraction of the optimal revenue with $n$ bidders is obtained by a second price auction with $n'$ bidders.

2. A simple auction (either selling the items separately or a second price auction with an entry fee) with $n'$ bidders generates $c$ times more revenue than the optimal auction with $n$ bidders.
Theorem 1

Let $\epsilon > 0$ and $n' = O(n/\epsilon)$. At least one of the following hold:

1. A $(1 - \epsilon)$-fraction of the optimal revenue with $n$ bidders is obtained by a second price auction with $n'$ bidders.

2. Any auction that guarantees a constant approximation to the optimal revenue with $n'$ bidders generates $c$ times more revenue than the optimal auction with $n$ bidders.
Theorem 1
Let $\epsilon > 0$ and $n' = O(n/\epsilon)$. At least one of the following hold:

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2. Any auction that guarantees a constant approximation to the optimal revenue with $n'$ bidders generates $c$ times more revenue than the optimal auction with $n$ bidders.

Note: for any auction (or best of a group of auctions) to get near optimal revenue with constant enhance competition, it is necessary for the auction to guarantee a constant fraction of the optimal revenue. Our result can be viewed as saying this is sufficient as well.
Our Results for Regular Distributions

We will propose a prior independent auction that generates almost optimal revenue with constant enhanced competition.
Our Results for Regular Distributions

We will propose a **prior independent auction** that generates almost optimal revenue with constant enhanced competition.

**Theorem 2**

Let $\epsilon > 0$ and $n' = O(n/\epsilon^2)$. When the items are regular, at least one of the following hold:

1. A $(1 - \epsilon)$-fraction of the optimal revenue with $n$ bidders is obtained by a second price auction with $n'$ bidders.

2. A prior-independent second price auction with an entry fee with $n'$ bidders generates $\frac{1}{\epsilon}$ times more revenue than the optimal auction with $n$ bidders.
Our Results for Regular Distributions

**Theorem 2**

Let $\epsilon > 0$ and $n' = O(n/\epsilon^2)$. When the items are regular, at least one of the following hold:

1. A $(1 - \epsilon)$-fraction of the optimal revenue with $n$ bidders is obtained by a second price auction with $n'$ bidders.
2. A prior-independent second price auction with an entry fee and $n'$ bidders generates $\frac{1}{\epsilon}$ times more revenue than the optimal auction with $n$ bidders.

**Hybrid Auction:**

- runs second price auction w.p. $1 - \epsilon$
- runs prior-independent second price auction with an entry fee w.p. $\epsilon$
Our Results for Regular Distributions

**Theorem 2**

Let $\epsilon > 0$ and $n' = O(n/\epsilon^2)$. When the items are regular, at least one of the following hold:

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2. A prior-independent second price auction with an entry fee and $n'$ bidders generates $\frac{1}{\epsilon}$ times more revenue than the optimal auction with $n$ bidders.

**Hybrid Auction:**

- runs second price auction w.p. $1 - \epsilon$
- runs prior-independent second price auction with an entry fee w.p. $\epsilon$

The hybrid auction with $n' = O(n/\epsilon^2)$ bidders obtains $(1 - \epsilon)^2$-fraction of the optimal revenue with $n$ bidders.
Theorem 1

Let \( \epsilon > 0 \) and \( n' = O(n/\epsilon) \). At least one of the following hold:

1. A \((1 - \epsilon)\)-fraction of the optimal revenue with \( n \) bidders is obtained by a second price auction with \( n' \) bidders.

2. A simple auction with \( n' \) bidders generates more revenue than the optimal revenue with \( n \) bidders.
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Event A: case (1) does not hold

Assuming event A, we prove:
Theorem 1

Let $\epsilon > 0$ and $n' = O(n/\epsilon)$. At least one of the following hold:

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Assuming event A, we prove:

• **Step one:** The optimal welfare with $n'$ bidders is much larger than the optimal welfare with $n$ bidders.
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Event A: case (1) does not hold

Assuming event A, we prove:

- **Step one:** The optimal welfare with \( n' \) bidders is much larger than the optimal welfare with \( n \) bidders.
- **Step two:** The optimal virtual welfare with \( n' \) bidders is much larger than the optimal virtual welfare with \( n \) bidders.
Theorem 1 Proof Outline

Theorem 1
Let $\epsilon > 0$ and $n' = O(n/\epsilon)$. At least one of the following hold:

1. A $(1 - \epsilon)$-fraction of the optimal revenue with $n$ bidders is obtained by a second price auction with $n'$ bidders.
2. A simple auction with $n'$ bidders generates more revenue than the optimal revenue with $n$ bidders.

Event A: case (1) does not hold

Assuming event A, we prove:

- **Step one:** The optimal welfare with $n'$ bidders is much larger than the optimal welfare with $n$ bidders.
- **Step two:** The optimal virtual welfare with $n'$ bidders is much larger than the optimal virtual welfare with $n$ bidders.
- **Step three:** Use connection between optimal revenue and virtual welfare.
Does welfare grow with number of bidders?

**Lemma**

Assume event A (second price auction with $n' = \frac{20n}{\epsilon}$ bidders extract at most $(1 - \epsilon)$ fraction of optimal revenue with $n$ bidders),

then (welfare with $n' = \frac{20n}{\epsilon}$ bidders) $\geq 20 \cdot$ (welfare with $n$ bidders).
**Lemma**

Assume event A (second price auction with \( n' = \frac{20n}{\epsilon} \) bidders extract at most \((1 - \epsilon)\) fraction of optimal revenue with \( n \) bidders),

then (welfare with \( n' = \frac{20n}{\epsilon} \) bidders) \( \geq 20 \cdot \) (welfare with \( n \) bidders).

\[
\text{First } n \text{ bidders} \quad 1 \text{ item case}
\]

\[
n' = \frac{20n}{\epsilon} \text{ bidders}
\]
Does welfare grow with number of bidders?

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First $n$ bidders

1 item case

$E[welfare with n'] \leq \frac{\epsilon}{20} \cdot E[welfare with n] + (1 - \frac{\epsilon}{20}) \cdot E[2nd highest value with n']$
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1 item case

\( n' = \frac{20n}{\epsilon} \) bidders

\[
\mathbb{E}[\text{welfare with } n] \\
\leq \frac{\epsilon}{20} \cdot \mathbb{E}[\text{welfare with } n'] + (1 - \frac{\epsilon}{20}) \cdot \mathbb{E}[\text{2nd highest value with } n'] \\
\leq \frac{\epsilon}{20} \cdot \mathbb{E}[\text{welfare with } n'] + (1 - \epsilon) \mathbb{E}[\text{welfare with } n]
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\[
\begin{align*}
\mathbb{E}[\text{welfare with } n] & \leq \frac{\epsilon}{20} \cdot \mathbb{E}[\text{welfare with } n'] + (1 - \frac{\epsilon}{20}) \cdot \mathbb{E}[\text{2nd highest value with } n'] \\
& \leq \frac{\epsilon}{20} \cdot \mathbb{E}[\text{welfare with } n'] + (1 - \epsilon) \cdot \mathbb{E}[\text{welfare with } n] \\
& \Rightarrow \epsilon \cdot \mathbb{E}[\text{welfare with } n] \leq \frac{\epsilon}{20} \cdot \mathbb{E}[\text{welfare with } n']
\end{align*}
\]
Theorem 1 Proof Outline

Theorem 1
Let $\epsilon > 0$ and $n' = O(n/\epsilon)$. At least one of the following hold:

1. A $(1 - \epsilon)$-fraction of the optimal revenue with $n$ bidders is obtained by a second price auction with $n'$ bidders.
2. A simple auction with $n'$ bidders generates more revenue than the optimal revenue with $n$ bidders.

Event A: case (1) does not hold

Assuming event A, we prove

- **Step one**: The optimal welfare with $n'$ bidders is much larger than the optimal welfare with $n$ bidders.
- **Step two**: The optimal virtual welfare with $n'$ bidders is much larger than the optimal virtual welfare with $n$ bidders.
- **Step three**: Use connection between optimal revenue and virtual welfare.
For any fixed number of bidders $N$, the optimal revenue is at most the expected virtual welfare, which is at most 8 times revenue from a simple auction (selling separately or second price with entry fee).
[CDW16]
For any fixed number of bidders \( N \), the optimal revenue is at most the expected virtual welfare, which is at most 8 times revenue from a simple auction (selling separately or second price with entry fee).

**Lemma Modified**
Assume event A (second price auction with \( n' = \frac{20n}{\epsilon} \) bidders extract at most \((1 - \epsilon)\) fraction of optimal revenue with \( n \) bidders), then (virtual welfare with \( n' = \frac{20n}{\epsilon} \) bidders) \( \geq 20 \cdot \) (virtual welfare with \( n \) bidders).
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Assume event A (second price auction with $n' = \frac{20n}{\epsilon}$ bidders extract at most $(1 - \epsilon)$ fraction of optimal revenue with $n$ bidders),

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**Proof of Theorem 1**

revenue from $n$ bidders $\leq$ virtual welfare from $n$ bidders
For any fixed number of bidders $N$, the optimal revenue is at most the expected virtual welfare, which is at most 8 times revenue from a simple auction (selling separately or second price with entry fee).

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Assume event A (second price auction with $n' = \frac{20n}{\epsilon}$ bidders extract at most $(1 - \epsilon)$ fraction of optimal revenue with $n$ bidders), then (virtual welfare with $n' = \frac{20n}{\epsilon}$ bidders) $\geq 20 \cdot$ (virtual welfare with $n$ bidders).

**Proof of Theorem 1**

revenue from $n$ bidders $\leq$ virtual welfare from $n$ bidders $\leq \frac{1}{20} \cdot$ virtual welfare from $n'$ bidders
[CDW16] For any fixed number of bidders $N$, the optimal revenue is at most the expected virtual welfare, which is at most 8 times revenue from a simple auction (selling separately or second price with entry fee).

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**Proof of Theorem 1**

revenue from $n$ bidders $\leq$ virtual welfare from $n$ bidders 
\[ \leq \frac{1}{20} \cdot \text{virtual welfare from } n' \text{ bidders} \]
\[ \leq \frac{8}{20} \cdot \text{revenue from a simple auction with } n' \text{ bidders} \]
Theorem 1
Let $\epsilon > 0$ and $n' = O(n/\epsilon)$. At least one of the following hold:

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Does virtual value grow with number of bidders?

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Assume event A (second price auction with \( n' = \frac{20n}{\epsilon} \) bidders extract at most \((1 - \epsilon)\) fraction of optimal revenue with \( n \) bidders), then

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**Virtual value from [CDW16]:**

\[
\Phi_{j}^{n}(v_{i}, v_{-i}) = \begin{cases} 
\tilde{\phi}_{j}(v_{i,j})^{+} & \text{if bidder } i \text{ gains the highest (and non-negative) utility from item } j \text{ in second price auction} \\
v_{i,j} & \text{otherwise}
\end{cases}
\]
Does virtual value grow with number of bidders?

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Virtual value from [CDW16]:

$$\Phi_j^n(v_i, v_{-i}) = v_{i,j} \cdot \mathbb{1}(v_i \notin R_j^{v_{-i}}) + \tilde{\phi}_j(v_{i,j})^+ \cdot \mathbb{1}(v_i \in R_j^{v_{-i}}).$$
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\]

Conditions:

- The virtual values must be at most the corresponding values.
- The distribution of virtual values for different bidders are independent and identical.
- The distribution of virtual values does not depend on the number \( n \) of bidders participating in the auction.
Does virtual value grow with number of bidders?

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Assume event A (second price auction with \( n' = \frac{20n}{\epsilon} \) bidders extract at most \((1 - \epsilon)\) fraction of optimal revenue with \( n \) bidders), then

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\[
\Phi_j^n(v_i, v_{-i}) = v_{i,j} \cdot 1(v_i \notin R_j^{v_{-i}}) + \tilde{\varphi}_j(v_{i,j})^+ \cdot 1(v_i \in R_j^{v_{-i}}).
\]

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\]

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Redefining Virtual Value

Idea: take the expectation (draw $n - 1$ ghost bidders)
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A modified virtual value:

$$\Phi^n_j(v_i) = \mathbb{E}_{v_{-i} \sim D_{n-1}} [\Phi^n_j(v_i, v_{-i})]$$
Redefining Virtual Value

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$$
\Phi^n_{j}(v_i) = v_{i,j} \cdot \mathbb{E}_{v_{-i} \sim D^{n-1}} \left[ 1 \left( v_i \notin R_{j}^{v_{-i}} \right) \right] + \tilde{\phi}_j(v_{i,j})^+ \cdot \mathbb{E}_{v_{-i} \sim D^{n-1}} \left[ 1 \left( v_i \in R_{j}^{v_{-i}} \right) \right].
$$
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$$
\Phi_j^n(v_i) = v_{i,j} \cdot \mathbb{E}_{v_{-i}\sim D^{n-1}} [ \mathbb{1} (v_i \notin R_j^{v-i}) ] + \tilde{\phi}_j(v_{i,j})^+ \cdot \mathbb{E}_{v_{-i}\sim D^{n-1}} [ \mathbb{1} (v_i \in R_j^{v-i}) ].
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Idea: fix some large $n' > n$, draw $n' - 1$ bidder values $w_i \sim \mathcal{D}^{n'-1}$
Redefining Virtual Value

Idea: fix some large $n' > n$, draw $n' - 1$ bidder values $w_{-i} \sim \mathcal{D}^{n'-1}$

A modified virtual value:

$$
\Phi^n_j(v_i) = v_{i,j} \cdot \mathbb{E}_{w_{-i} \sim \mathcal{D}^{n'-1}} \mathbb{I}(w_i \notin R_j^{w_{-i}}) + \tilde{\phi}_j(v_{i,j})^+, \mathbb{E}_{w_{-i} \sim \mathcal{D}^{n'-1}} \mathbb{I}(w_i \in R_j^{w_{-i}}).
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Idea: fix some large \( n' > n \), draw \( n' - 1 \) bidder values \( w_{-i} \sim \mathcal{D}^{n' - 1} \)

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Conclusion

We show that, for all $m$ and $n$, an arbitrarily large constant fraction of the optimal revenue from selling $m$ items to $n$ bidders can be obtained via simple auctions with $O(n)$ bidders.

Future directions:
• Obtains full optimal revenue with $O(n)$ bidders?
• Obtain almost optimal revenue with $n + o(n)$ bidders or prove a lower bound?
• Our work can also be viewed as proving for additive valuations an equivalence between auctions that gets a constant fraction of the optimal revenue and auctions that has $O(n)$ enhanced competition. Can we prove this for more general class of valuation functions?
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Thank you!

Questions?


